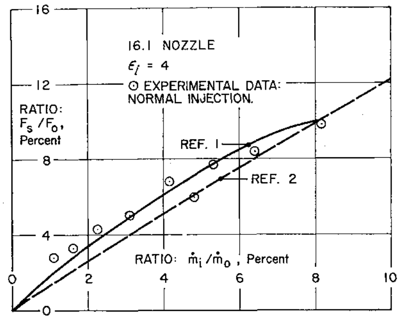

Fig. 1 Side force vs injection mass flow rate (16.1 nozzle, ϵ_i = 4).

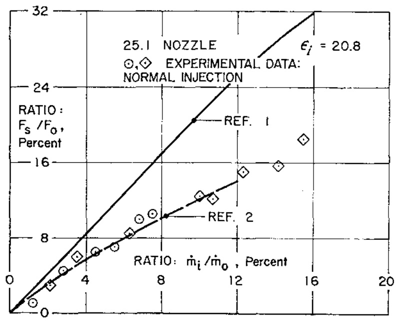

Fig. 2 Side force vs injection mass flow rate (25.1 nozzle, ϵ_i = 20.8).

flow. The existence of shock structure was ignored, and no mixing of the primary and secondary fluids was considered. To compute primary thrust, the secondary flow was assumed to fill a segment of an annulus at the exit, and the static pressure and pressure gradient in the axial direction were assumed uniform at the exit station. The primary thrust with injection was then computed by assuming spherical source flow at the exit and integrating over the primary area. The secondary thrust was computed by making a contradictory assumption that the primary flow provides an aerodynamic nozzle for the secondary flow. The vector sum of the two computations gave thrust magnitude and direction with injection.

Figures 1 and 2 duplicate Figs. 3 and 9 of Ref. 1, with the addition of computed curves based on the equations of Ref. 2 (or Ref. 3). It should be noted that the computations based on Ref. 2 are for the small injection port area, whereas the data are for both large and small injection port area. However, the difference in the computations for the two areas is less than the data scatter, and so only one curve is shown.

In Fig. 1, data agree closely with Ref. 1, whereas in Fig. 2, the best agreement is with Ref. 2. Of the other figures of Ref. 1 (not shown here), the data are intermediate between the two theoretical curves for Figs. 4, 5, and 8, and agree closely with Ref. 2 in Fig. 6 and with Ref. 1 in Fig. 7.

The relative agreement of the two integral methods, in spite of the enormous differences in assumptions, is a compelling argument for the power of the integral technique in problems of this kind. Although neither method is completely satisfactory, it seems quite certain that, with experimental investigation of the exit plane flow field to suggest more appropriate assumptions, an integral technique could be developed which would accurately predict the effect of secondary injection in a rocket nozzle.

References

[1] Karamcheti, K. and Hsai, H. T. S., "Integral approach to an approximate analysis of thrust vector control by secondary injection," AIAA J. 1, 2538–2544 (1963).

[2] Simon, W. E., "A theoretical model for thrust vector control with gas injection," IAS Sherman M. Fairchild Fund Paper FF-35 (January 21–23, 1963).

[3] Simon, W. E., "A theoretical model for thrust vector control with gas injection" (with classified data comparison), 19th Annual JANAF-ARPA-NASA Solid Propellant Meeting Bull., Applied Physics Lab., Johns Hopkins Univ. (July 1963); confidential.

Reply by Authors to W. Simon

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WE are delighted to see that the ideas expressed in our paper are worthy of re-emphasis. In assessing the utility of the actual relations given in our paper, the various approximations and restrictions described therein should be borne in mind.

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Errata: "Variable-Mesh Difference Equation for the Stream Function in Axially Symmetric Flow"

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[AIAA J. 2, 163–164 (1964)]

EDWARD F. Blick of the University of Oklahoma has apprised the author of two errors in a recent paper. The third term in Eq. (4) should be

$$2\psi(z, r)/k^2\alpha$$

Similarly, the third term in Eq. (6) should be

$$2\psi(z, r)/h^2\beta$$

The errors did not carry over to the final result, Eq. (7).

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Conical Shock-Wave Angle

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IN Ref. 1, Zumwalt and Tang have developed the approximate expression

$$\sin^2\theta_w = (1/M^2) + 0.038(\theta_s/10)^{1.87} \qquad (1)$$

relating the shock-wave (half) angle θ_w to the freestream Mach number M and the cone (half) angle θ_s, in degrees, for the axially symmetric flow of air past a cone with an attached shock wave.

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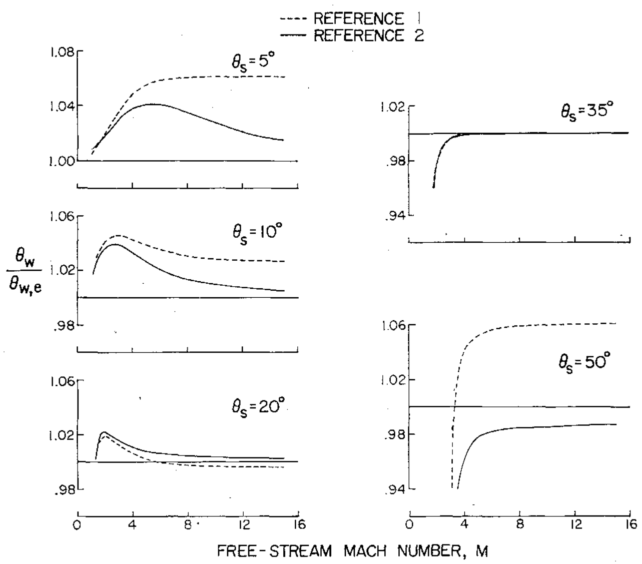

Fig. 1 Ratio of calculated shock-wave (half) angle θ_w [from Eqs. (1) and (2)] to exact value $\theta_{w,e}$ (from Ref. 3), as function of freestream Mach number M, for various cone angles (γ = 1.405).

It is of interest to compare the approximate values of θ_w calculated from Eq. (1) with those provided by the writer's approximation [Ref. 2, Eq. (17)];

$$\sin^2\theta_w = (1/M^2) + [(\gamma + 1)/2] \sin^2\theta_s \qquad (2)$$

where γ denotes the ratio of specific heats of the gas, in the present case, air. The necessary calculations have been carried out for cone angles ranging from $\theta_s = 5°$ to $\theta_s = 50°$, and the results are presented in Fig. 1.

The mathematical and physical bases for the approximate expression (2) have been adequately described in Ref. 2. Through the foregoing Eq. (2), these provide an explanation of the Mach number independence of the conical shock pressure coefficient described in Ref. 1.

Equation (2) and other results of Ref. 2 have recently been derived by South[4] as an approximate, equilibrium solution resulting from the application of a technique in differential equations to the analysis of flows with vibrational relaxation. It should be noted that Eq. (2) was obtained heuristically by Simon and Walter [cf. Eq. (13), Ref. 5].

References

[1] Zumwalt, G. W. and Tang, H. H., "Mach number independence of conical shock pressure coefficient," AIAA J. 1, 2389–2391 (1963).

[2] Hord, R. A., "An approximate solution for axially symmetric flow over a cone with an attached shock wave," NACA TN 3485 (October 1955).

[3] Staff of the Computing Section, Center of Analysis (under direction of Z. Kopal), "Tables of supersonic flow around cones," TR 1, Massachusetts Institute of Technology (1947).

[4] South, J. C., Jr., "Application of Dorodnitsyn's integral method to nonequilibrium flows over pointed bodies," NASA TN D-1942 (August 1963).

[5] Simon, W. E. and Walter, L. A., "Approximations for supersonic flow over cones," AIAA J. 1, 1696–1698 (1963).